# Optimizing EPR Allocation Using Double Auction in Quantum Networks

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Abstract—In quantum networks, the efficient allocation of EPR pairs is a crucial factor that affects the reliability and efficiency of communication. However, generating and maintaining EPR pairs incurs costs, making it a challenge to optimally allocate the limited resources. Existing research focuses on scenarios in which a single EPR provider allocates resources within one system. However, as quantum computing becomes increasingly common, multiple EPR providers are expected to act as the current ISPs of quantum networks to provide a vast internet of working quantum communication. This change in environment will cause existing methods of EPR allocation to become uneven and biased. This study proposes an optimal allocation method for EPR pairs using double auction. By utilizing the market mechanism of double auction, the allocation of EPR pairs can be conducted fairly, achieving a distribution that does not bias towards specific nodes, and enabling efficient allocation and dynamic operation based on supply and demand. This paper applies and evaluates the double auction method to the problem of EPR pair allocation in quantum networks.

### I. INTRODUCTION

In the future quantum internet, EPR providers specializing in the supply of EPR pairs are expected to emerge. The role of EPR providers will be analogous to that of ISPs in the current internet, and research on systems and quantum hardware assuming such providers is actively being conducted [1]. However, [2] indicates that existing algorithms for resource allocation in quantum networks focus on maximizing the number of allocations, leading to potential biases. Therefore, we propose a mechanism to efficiently and fairly distribute large-scale EPR allocations in future quantum network environments.

This paper proposes an optimal allocation method for EPR pairs using double auction. Double auction is a market mechanism where buyers and sellers simultaneously place bids, determining the optimal price and transaction volume. By utilizing the market mechanism of aouble auction, the allocation of EPR pairs can be conducted fairly, achieving a distribution that does not bias towards specific nodes, and enabling efficient allocation and dynamic operation based on supply and demand.

# II. QUANTUM NETWORK

# A. EPR Pairs

Quantum Networks transfer data from one location to another through process called quantum teleportation. This allows the transmission of quantum bit states to remote locations without directly transferring the quantum bits themselves. Howver, each act of quantum teleportation consume an EPR pair with a high fidelity, which refers to the quality of entangled state of an EPR pair. Hence, an efficient allocation of EPR resources is crucial to the quality of a quantum network.

# B. EPR Providers

In quantum networks, entities responsible for providing entanglement resources (EPR pairs) exist within each system. In this paper, we refer to providers that distribute EPR across multiple different systems as EPR providers. These providers generate, maintain, and distribute EPR pairs, supporting quantum communication between nodes within the network. The efficient operation of EPR providers significantly impacts the overall performance of the quantum network.

# III. DOUBLE AUCTION

Auctions are widely used as a means to achieve efficient trading and there are multiple examples of its application on resource allocation in MEC (Mobile Edge Computing) [3][7] and cloud computing [6].

# A. Truthful Bidding

The VCG (Vickrey-Clarke-Groves) mechanism is an auction mechanism used to maximize social welfare and achieve efficient resource allocation. Participants have an incentive to report their true valuations (truth-telling is a dominant strategy). Participants report their valuations, and based on these, the allocation that maximizes social welfare (total utility) is determined. Each participant's payment is calculated as the difference between the social welfare assuming only other participants exist and the social welfare when the participant is included. This corresponds to "internalizing externalities," where each participant pays based on the impact they have on others. The VCG mechanism induces truth-telling as a dominant strategy, making strategic reporting unnecessary.

# IV. PROPOSING SYSTEM

In this paper, we propose a quantum resource allocation system using TCDA (Truthful Combinatorial Double Auction) where multiple EPR providers and multiple users seek EPR pairs across multiple systems.

#### A. Data Process

To simulate a quantum network environment, we use a quantum network simulator called Netsquid [5] to calculate Fidelity based on geographical distance, treating Fidelity as a cost in the auction, and allocate resources using TCDA.

The platform receives payments from users and provides EPR supplied by EPR providers to the users. The allocation algorithm uses a padding method based on linear programming to optimize the resource allocation.

#### V. EVALUATION

# A. Evaluation Method

To evaluate the proposed method, we compare it with other commonly used algorithms.

FIFO (First Come First Out) matches EPR Providers with the first available user in the auction to ensure the maximum amount of trades. Random selects a seller randomly from those that meet the conditions for the Buyer, and proportional allocation selects the conditional sellers while dissuading the participants from overbidding. If a user i requests  $d_i$  and the total requested amount is D (sum of all user requests) and R is the total amount of resource, their allocated share  $a_i$  is:

$$a_i = \frac{d_i}{D} \times R \tag{1}$$

To evaluate the efficiency of the EPR allocation results, we measure the the utility of each participant to calculate the total social welfare.

The true valuation refers to the actual value or cost that participants report accurately during the bidding or selling process in the auction. The utility  $U_i^b$  of user (Buyer) i is the value of EPR  $v_i$  minus the payment  $p_i^b$ . The utility  $U_i^s$  of EPR provider (Seller) j is the reward  $r_i^s$  received from the user minus the cost  $c_i$  of providing EPR. The utility  $U^p$  of the platform (Auctioneer) is the difference between the total payment from all users and the total reward to all EPR providers. Let  $x_i$  be a binary variable that takes 1 if EPR resources are allocated to user i by the auction and 0otherwise. Similarly, let  $y_j$  be a binary variable that takes 1 if the EPR provided by EPR provider j is allocated to a user and 0 otherwise. Then,  $U_i^b$ ,  $\hat{U}_i^s$ , and  $\hat{U}^p$  are obtained by the following equations.

$$U_i^b = x_i v_i - p_i^b (2)$$

$$U_i^s = r_j^s - y_j c_j \tag{3}$$

$$U_{i}^{b} = x_{i}v_{i} - p_{i}^{b}$$

$$U_{i}^{s} = r_{j}^{s} - y_{j}c_{j}$$

$$U^{p} = \sum_{i \in I} p_{i}^{b} - \sum_{i \in I} r_{j}^{s}$$
(2)
(3)

# B. Evaluation Results

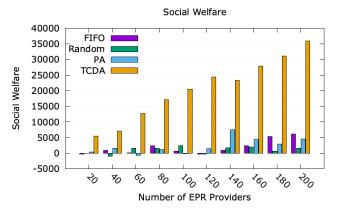


Fig. 1. Social Welfare Evaluation

Fig. 1 plots the SW of each method, and Fig. 2 plots the number of EPR allocated users for each method against the

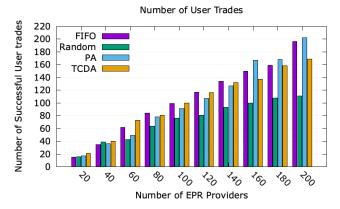


Fig. 2. Number of Successful User Trades

number of EPR providers. The proposed method manages to maintain the number of transactions within a similar level with the other existing methods while significantly increasing the aggregate social welfare of the entire allocation process. The proposed method also prevents the utility of users and EPR providers from becoming negative. This promotes rational transactions for each user and EPR provider, ensuring overall fairness. This difference arises from the mechanism that guarantees budget balance under specific price determination conditions and the individual rationality and budget balance ensured through truthful bidding and selling strategies.

# VI. CONCLUSION

In this paper, we proposed and evaluated an auction system that efficiently allocates EPR and conducts fair transactions in a changing environment due to the spread of quantum networks. By using auction methods, we confirmed that rational allocation based on market principles can be achieved, and the total utility of the platform can be secured compared to existing allocation methods.

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