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Incentivizing Users for High Quality of Crowdsensing Data

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Abstract Mobile crowdsensing has become a popular paradigm for collecting data from smart device users. Traditional research mainly considered on designing incentive mechanism based on users' effort to provide sensing data without considering users' ability to obtain the data. In this paper, users' ability to obtain the data has been considered to maximize the crowdsensing platform's data quality when designing the price based incentive mechanism for users. Our proposed optimal pricing based incentive mechanism is validated by numerical simulations.

Keywords Mobile crowdsensing, data quality, network economics, convex optimization

1. Introduction

Internet of Things (IoT) services are expected to enrich life of people through utilizing various sensing data obtained by many IoT devices [1]. IoT services consist of IoT devices, IoT platforms, and service consumers. IoT devices measure various environmental data, e.g., temperature, humidity, and hazardous substances concentration, and provide the sensed data to the IoT platforms. After analyzing the collected data, the IoT platforms extract various information and knowledges which are useful to consumers, and provide various services utilizing the obtained information and knowledges to service consumers.

Sensing functions measuring various environmental data are provided to many smartphones recently. Because of proliferation of smartphones with embedded sensors, mobile users become active contributors providing various kinds of sensing data, and *mobile crowdsensing* has attracted a wide attention as a new scheme, in which smartphones is seen as a kind of IoT devices [2]. However, smartphone users are not under control of the IoT platforms, and they selfishly determine how much effort they make on acquiring the sensing data by smartphones. To collect sensing data from smartphone users, the IoT platforms are required to pay incentives to smartphone users, and many current commercial crowdsensing systems, e.g., Amazon Mechanical Turk, post a fixed price to smartphone users to incentivize participation [3]. However, the quality of sensing data will depend on the effort of smartphone users in crowdsensing, so the effort of smartphone users should be reflected on the incentive paid to smartphone users to give them a motivation to make higher efforts on acquiring sensing data.

By reflecting the quality of sensing data on the price paid to smartphone users, IoT platforms can expect smartphone users to make more effort on improving the quality of sensing data. Therefore, some existing works proposed to give smartphone users incentivizes depending on the data value, i.e., quality of data [4][5][6]. The quality of sensing data depends on the ability of smartphone users in sensing data, so the achieved quality is different among smartphone users even when they make the same effort. The IoT platforms can expect to collect sensing data with high quality by smaller amount of payment if they pay the incentive to smartphone users with higher ability to acquire precise data. The total amount of incentives which the IoT platforms can afford paying to smartphone users is limited, so the IoT platforms need to carefully select smartphone users to which incentives are paid to maximize the total quality of sensing data collected under the budget constraint of the IoT platforms. However, existing incentive mechanisms did not consider the ability of smartphone users on sensing data as well as the constraint of the total budget of IoT platform.

In this paper, we propose a new incentive mechanism for smartphone users to provide precise sensing data to the IoT platforms by formulating a data quality maximization problem. The optimal solution is obtained and the numerical evaluation has proved the effectiveness of our work.

2. Related Works

There are some existing works proposed to use prices paid to smartphone users to incentivize them to provide sensing data. Lee et al. proposed a reverse auction model for setting prices [7], and Jin et al. also proposed a double auction model directly matching IoT devices and service consumers [8][9]. However, the IoT platforms cannot maximize the value of data collected from the smartphone users because smartphone users do not know the quality of data they provide to the IoT platforms. Han et al. considered the effort of smartphone users in sensing data by setting the price posted to smartphone users so that the cost of IoT platforms, i.e., the total amount of incentive paid to smartphone users, was minimized when the distribution of quality of data can be estimated from the efforts of smartphone users [3]. However, the fixed price was posted to all smartphone users, so the IoT platforms cannot incentivize smartphone users to make higher effort to get sensing data.

To incentive smartphone users to make higher effort on acquiring sensing data with higher quality, the price paid to each smartphone user should depend on the quality of sensing data provided to the IoT platforms. Some existing works proposed to set the price based on the value of sensing data. For example, Niyato et al. proposed to set the fee obtained from service consumers based on the value of sensing data which was defined by the detection probability of event [10]. However, this method cannot be applied for setting the incentive paid to IoT devices, i.e., smartphone users.

Peng et al. proposed to set the incentive based on the estimated quality of sensing data provided by each smartphone user, so smartphone users were motivated to make effort to improve the quality of sensing data [5]. Jin et al. also proposed to set the incentives paid to smartphone users so that the profit of IoT platforms was maximized when smartphone users maximized their utility by optimizing the variance of error distribution of sensing data which was assume to obey the Gauss distribution [4]. Moreover, Zheng et al. also proposed to set the incentives paid to smartphone users based on the accuracy of sensing data which was estimated by Gaussian Process [6]. However, these methods did not consider both the ability of smartphone users to acquire sensing data and the budget of IoT platforms, so the IoT platforms cannot maximize the total quality of sensing data with the budget constraint.

3. System model

In this paper, we only consider one specific location l. The smartphone users' set at location l is denoted as $\mathcal{N}_l = \{1, ..., N_l\}$. $\mathcal{N}_l^* \subseteq \mathcal{N}_l$ denotes the smartphone users' set in which smartphone users provide data for the IoT platform at location l. The quality of data is defined as follows in Eq.(1).

$$q_i(s_i) = \gamma_i \ln(1 + s_i) \tag{1}$$

where γ_i is the reputation of smartphone user $i \in \mathcal{N}_l$, which is recorded by IoT platform. s_i is smartphone user i' effort to generate data.
The cost of data is defined as follows in Eq.(2).

$$c_i(s_i) = \lambda_i s_i \tag{2}$$

where λ_i the cost parameter per unit effort.
smartphone users' utility is defined as follows in Eq.(3).

$$\begin{aligned} u_i(s_i) &= p_i q_i(s_i) - c_i(s_i) \\ &= p_i \gamma_i \ln(1 + s_i) - \lambda_i s_i \end{aligned} \tag{3}$$

where p_i is the price per unit data quality paid from IoT platform. $p_i q_i(s_i)$ is the payment by IoT platform, which is proportional to smartphone users' data quality. Please note that IoT platform sets the payment from its budget B_l. Given the payment from IoT platform, smartphone user i tries to maximize his/her utility by setting suitable s_i. Therefore, a utility maximization problem for smartphone users is formulated as follows.

$$\begin{aligned} &\max_{s_i} u_i(s_i) \\ &\text{s.t.} \quad s_i \leq 0 \end{aligned} \tag{4}$$

Proposition 1. *The solution for problem in Eq.(4) is as follows.*

$$s_i^* = \frac{\gamma_i}{\lambda_i} p_i - 1 \tag{5}$$

Proof: Since $\frac{d^2 u_i}{ds_i^2} = -\frac{p_i \gamma_i}{(1+s_i)^2} < 0$, then optimal solution exists for problem defined in Eq.(4). By setting $\frac{du_i}{ds_i} = 0$, the optimal solution can be obtained as follows.

$$s_i^* = \frac{\gamma_i}{\lambda_i} p_i - 1 \tag{6}$$

Q.E.D

4. Quality maximization problem

Given a budget B_l for location l, IoT platform tries to maximize the sum of data quality at location l. A quality maximization problem (QMP) is formulated as follows in Eq.(7).

$$\begin{aligned} &\max_{p_i} \sum_{i \in \mathcal{N}_l^*} q_i(s_i) \\ &\text{s.t.} \quad \sum_{i \in \mathcal{N}_l^*} p_i q_i(s_i) \leq B_l \\ &\quad u_i \geq 0, \forall i \in \mathcal{N}_l \\ &\quad p_i \geq 0, \forall i \in \mathcal{N}_l \end{aligned} \tag{7}$$

substitue $s_i^* = \frac{\gamma_i}{\lambda_i} p_i - 1$ to the objective function and constraint, we have the following new problem

$$\begin{aligned} &\max_{p_i} \sum_{i \in \mathcal{N}_l^*} q_i(s_i^*) \\ &= \max_{p_i} \sum_{i \in \mathcal{N}_l^*} \gamma_i \ln(\frac{\gamma_i}{\lambda_i} p_i) \\ &\text{s.t.} \quad \sum_{i \in \mathcal{N}_l^*} p_i q_i(s_i^*) = \sum_{i \in \mathcal{N}_l^*} p_i \gamma_i \ln(\frac{\gamma_i}{\lambda_i} p_i) \leq B_l \\ &\quad p_i \geq 0, \forall i \in \mathcal{N}_l^* \end{aligned} \tag{8}$$

Please note that the condition $u_i \geq 0$ in Eq.(7) is no longer needed in the new problem in Eq.(8) since when substitute the optimal s_i^* to the objective function, u_i is maximized.

5. Theoretic analysis

As for the objective function of the problem defined in Eq.(8), we have the following **Proposition 2**.

Proposition 2. *The objective function of the new problem in Eq.(8) is concave.*

Please refer to Appendix **A.1** for the proof of **Proposition 2**.

Definition 1. *The Lambert W function [11] is a set of functions that are the inverse of the function $f(z) = ze^z$, where e^z is the exponential function, and z is any complex number. We can express it as follows.*

$$z = f^{-1} = W(ze^z) \tag{9}$$

By substituting $z_0 = ze^z$ into the Eq.(9), we get the definition of the W function as follows.

$$z_0 = W(z_0) e^{W(z_0)} \tag{10}$$

Properties of Lambert W function [11] are as follows.

- Property (1): The exponential of Lambert W function is as follows.

$$e^{W(z)} = \frac{z}{W(z)} \tag{11}$$

- Property (2): The derivative of Lambert W function is as follows.

$$\frac{\mathrm{d}W}{\mathrm{d}z} = \frac{W(z)}{z(1+W(z))} \quad \text{for } z \notin \left\{0, -\frac{1}{e}\right\} \tag{12}$$

Lemma 1. *As for the following equation in Eq.(13), m and n are the constants, x is the variable.*

$$m = \frac{1}{x} \cdot \frac{1}{[\ln(nx)+1]} \tag{13}$$

Its solution is as follows.

$$x = \frac{e^{W(\frac{ne}{m})}}{ne} \tag{14}$$

or

$$x = \frac{1}{mW(\frac{ne}{m})} \tag{15}$$

*where $W(\cdot)$ is the Lambert W function defined in **Definition 1**.*

Please refer to Appendix **A.2** for the proof of **Lemma 1**.
The solution is illustrated in the following **Proposition 3**.

Proposition 3. *The solution is for Eq.(7) is $p_i^* = h_i'(\alpha^*)$, where function $h_i'(\alpha^*)$ is defined in Eq.(33),*

$$p_i^* = h_i'(\alpha) = \frac{\lambda_i e^{W(\frac{e\gamma_i}{\lambda_i \alpha})}}{e\gamma_i} \tag{16}$$

and α^* is determined by Eq.(42).

$$G(\alpha) = B_l \tag{17}$$

where

$$G(\alpha) = \sum_{i \in \mathcal{N}_l^*} \frac{\gamma_i [W(\frac{\gamma_i e}{\lambda_i \alpha}) - 1]}{\alpha[1 + W(\frac{\gamma_i e}{\lambda_i \alpha})]} + \sum_{i \in \mathcal{N}_l^*} \left[\frac{\gamma_i}{\alpha W(\frac{\gamma_i e}{\lambda_i \alpha})[1 + W(\frac{\gamma_i e}{\lambda_i \alpha})]} \ln(\frac{\gamma_i}{\lambda_i} \cdot \frac{1}{\alpha W(\frac{\gamma_i e}{\lambda_i \alpha})})\right] \tag{18}$$

Please refer to Appendix **A.3** for the proof of **Proposition 3**. It is difficult to get the explicit solution for α^*, we will get the optimal solution α^* as well as p_i^* by numeric analysis.

6. Numeric analysis

In this section, the performances of our optimal pricing based incentive mechanism are evaluated by comparing them with fixed pricing based incentive mechanism.

We have implemented a simulator by Python 2.7.1. The number of smartphone users N is assumed as 10, and the smartphone users' reputation parameter γ_i ($\forall i \in \mathcal{N}_l$) is extracted from uniform distribution U[10, 20]. smartphone users' cost parameter λ_i ($\forall i \in \mathcal{N}_l$) is extracted from uniform distribution U[2, 8].

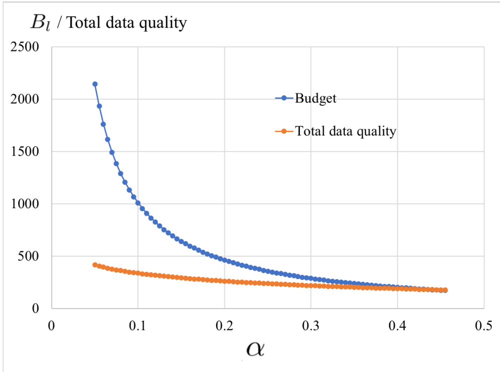

Fig. 1 Budget B_l and Total data quality vs. α.

Firstly, we find the optimal solution by numeric analysis. According to the theoretic analysis in section 5, to find the optimal Lagrangian multiplier α is important to solve the primary problem Eq.(4). It is difficult to get explicit solution for α by Eq.(42) We set α's value from 0.05 to 0.46 by step 0.005, and calculate $G(\alpha)$'s value by Eq.(42) (Please note that $G(\alpha)$'s value is equal to B_l). Fig. 1 shows how the budget B_l and total data quality changes with different α from from 0.05 to 0.46. Therefore, the optimal solution α^* can easily be gotten from Fig. 1. Once optimal α^* is given, the value of user i's optimal price p_i^*, optimal effort s_i^*, and corresponding data quality $q_i(s_i^*)$ can also be gotten from Eq.(33), Eq.(5), and Eq.(1).

Secondly, we compare our optimal pricing based incentive mechanism with fixed pricing based incentive mechanism. As for fixed pricing based incentive mechanism, the IoT platform pays for all the smartphone users with the same price. The methods to calculate for optimal effort and data quality are the same with that of optimal pricing based incentive mechanism. Fig. 2 shows the total data quality of optimal pricing based incentive mechanism and fixed pricing based incentive mechanism. Both of them increase with the budget B_l. The total data quality of our proposed optimal pricing based incentive mechanism are much higher. The reason behind this phenomena is that our proposed incentive mechanism sets different incentive for different smartphone users to maximize the data quality provided by smartphone user.

Fig. 3 show the comparison between the price of our

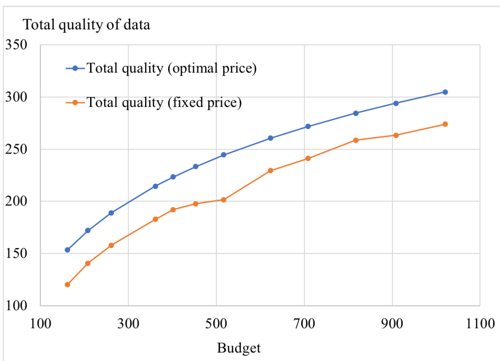

Fig. 2 Total data quality comparison.

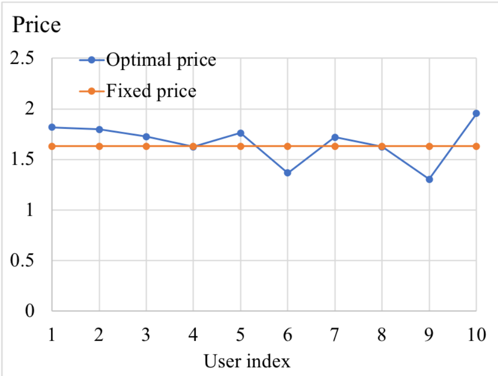

Fig. 3 Price comparison when B_l = 403.

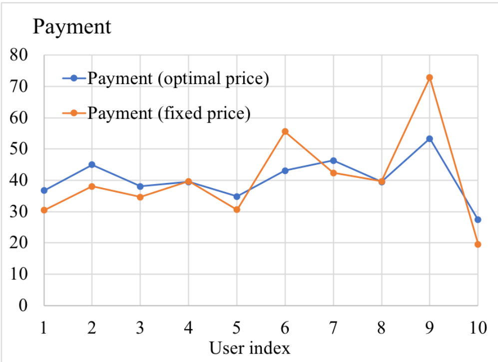

Fig. 4 Payment comparison when B_l = 403.

proposed incentive mechanism p_i ($\forall i \in \mathcal{N}_l$) and the fixed pricing incentive mechanism when the budget B_l is 403. It can be seen that the price of our proposed incentive mechanism varies with different smartphone users. Fig. 4 show the payment of IoT platform $p_i q_i(s_i)$ when the budget B_l is 403. The reason is that the fixed prices are higher than the optimal prices in our proposed mechanism as shown in Fig. 3 for user 6 and 9. Then, according to Eq. (5) in **Proposition** 1, user 6 and 9's effort is much higher. According to the payment formula $p_i q_i(s_i)$, the payment for user 6 and 9 are much higher than our proposed incentive mechanism as shown in Fig. 4. Our optimal pricing based incentive mechanism tries to maximize the total quality of sensing data as defined in Eq. (7) by allocating the budget B_l in global manner over different users, while the fixed pricing incentive mechanism does not consider the total quality of sensing data maximization.

Fig. 5 shows the data quality comparison. It shows the different data qualities of different smartphone users are different. And the data quality of our proposed incentive mechanism is much higher than that of fixed pricing incentive mechanism.

7. Conclusion

In this paper, we propose to set the price paid to smartphone users depending on the ability of acquiring sensing data which is estimated through the reputation of users to maximize the quality of data provided by the users. We formalize the data quality optimization problem under the platform's budget constraint, and the optimum

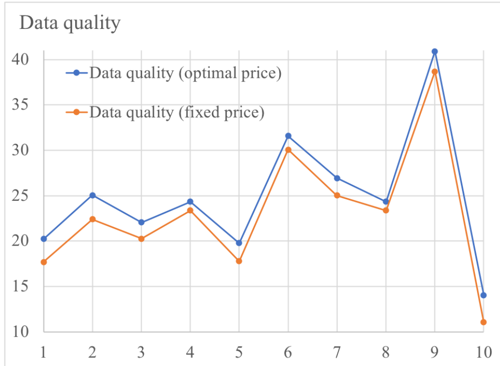

Fig. 5 Data quality comparison when B_l = 403.

price is analyzed by using the Lambert W function and Karush-Kuhn-Tucker (KKT) conditions. Numeric analysis validated our proposed incentive mechanism is effective and showed the the total data quality is maximized for the platform.

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Appendix

A.1 Proof of Proposition 2

We define a function as follows:

$$f_i(p_i) = \gamma_i \ln(\frac{\gamma_i}{\lambda_i} p_i) \quad (19)$$

The second-order differentiation of $f_i(p_i)$ is as follows.

$$\frac{d^2 f_i}{dp_i^2} = -\frac{\gamma_i}{p_i^2} \quad (20)$$

Because $\lambda_i > 0$, it is obvious that $\frac{d^2 f_i}{dp_i^2} < 0$. Thus, $f_i(p_i)$ is a concave function of p_i.

We define function the objective function of Eq.(8) as follows.

$$f(\boldsymbol{p}) = \sum_{i \in \mathcal{N}_l^*} \gamma_i \ln(\frac{\gamma_i}{\lambda_i} p_i) \quad (21)$$

where $\boldsymbol{p} = \{p_1, p_2, ..., p_{N_l^*}\}$ is a vector. Then we can express $f(\boldsymbol{p})$ by $f_i(p_i)$ as follows.

$$f(\boldsymbol{p}) = \sum_{i \in \mathcal{N}_l^*} f_i(p_i) \quad (22)$$

Since $f(\boldsymbol{p})$ is nonnegative weighted sums of concave functions $f_i(p_i)$ (see section 3.2.1 of book [**?**]), then $f(\boldsymbol{p})$ is concave.

A.2 Proof of Lemma 1

Proof: We define $y = \ln nx$, then $x = \frac{e^y}{n}$. By substituting $x = \frac{e^y}{n}$ to Eq.(13), we have the transformed one as folllows.

$$m = \frac{n}{e^y} \cdot \frac{1}{(y+1)} \quad (23)$$

or Eq.(23) can be expressed as follows.

$$e^{(y+1)}(y+1) = \frac{ne}{m} \quad (24)$$

According the definition of Lambert W function, we can have the solution y of Eq.(24) as follows.

$$y = W(\frac{ne}{m}) - 1 \quad (25)$$

Then x can be expressed as follows.

$$x = \frac{e^{[W(\frac{ne}{m})-1]}}{n} = \frac{e^{W(\frac{ne}{m})}}{ne} \quad (26)$$

According to property (1) of Lambert W function, x can also be expressed as follows.

$$x = \frac{e^{W(\frac{ne}{m})}}{ne} = \frac{\frac{\frac{ne}{m}}{W(\frac{ne}{m})}}{ne} = \frac{1}{mW(\frac{ne}{m})} \quad (27)$$

A.3 Proof of Proposition 3

Proof: The Lagrangian of the problem in Eq.(8) is as follows.

$$L(\boldsymbol{p}, \alpha, \beta) = -f(\boldsymbol{p}) + \alpha\Big[\sum_{i \in \mathcal{N}_l^*} p_i \gamma_i \ln(\frac{\gamma_i}{\lambda_i} p_i) - B_l\Big] - \sum_{i \in \mathcal{N}_l^*} \beta_i p_i \quad (28)$$

where $\alpha \geq 0$, $\beta_i \geq 0$, $\beta = (\beta_1, ..., \beta_{N_l^*})$ is the Lagrangian dual variable (or Lagrangian multiplier). The Karush-Kuhn-Tucker (KKT) conditions [**?**] for optimality are as follows.

$$\begin{cases} \text{Primal constraints:} \\ \quad \sum_{l \in \mathcal{N}_l^*} p_i - B_l \leq 0 \\ \quad p_i \geq 0 \\ \text{Dual constraints:} \\ \quad \alpha \geq 0 \\ \quad \beta_i \geq 0, \forall i \in N_l^* \\ \text{Complementary slackness:} \\ \quad \alpha(\sum_{i \in \mathcal{N}_l^*} p_i \gamma_i \ln(\frac{\gamma_i}{\lambda_i} p_i) - B_l) = 0 \\ \quad \beta_i p_i = 0, \forall i \in \mathcal{N}_l^* \\ \text{Gradient of Lagrangian with respect to } p_i \text{ vanishes:} \\ \quad \nabla L(\boldsymbol{p}, \alpha, \beta) = 0 \end{cases} \quad (29)$$

$$\nabla L(\boldsymbol{p}, \alpha, \beta) = -\frac{\gamma_i}{p_i} + \alpha\gamma_i[\ln(\frac{\gamma_i}{\lambda_i} p_i) + 1] - \beta_i = 0 \quad (30)$$

From Eq.(30), we have

$$\frac{\gamma_i}{p_i} = \alpha\gamma_i[\ln(\frac{\gamma_i}{\lambda_i} p_i) + 1] - \beta_i \quad (31)$$

It is obvious that $p_i > 0$, then $\beta_i = 0$, we have

$$\alpha = \frac{1}{p_i^*} \cdot \frac{1}{[\ln(\frac{\gamma_i}{\lambda_i} p_i^*) + 1]} \quad (32)$$

Eq.(32) has the same structure of Eq.(13) in **Lemma 1** if we assume that $m = \alpha$, and $n = \frac{\gamma_i}{\lambda_i}$. The solution of p_i^* can be easily obtained by **Lemma 1** as follows.

$$p_i^* = h_i'(\alpha) = \frac{\lambda_i e^{W(\frac{e\gamma_i}{\lambda_i \alpha})}}{e\gamma_i} = \frac{1}{\alpha W(\frac{\gamma_i e}{\lambda_i \alpha})} \quad (33)$$

where $h_i'(\alpha)$ is the function used to express p_i^*. By substituting property (2) of Lambert W function, the derivative of $h_i'(\alpha)$ can be obtained as follows.

$$h_i'(\alpha) = -\frac{1}{\alpha^2[1 + W(\frac{\gamma_i e}{\lambda_i \alpha})]} \quad (34)$$

Since $\lambda_i > 0$, then $\frac{\lambda_i}{p_i} = \alpha > 0$. By complementary slackness, we have the following conditions.

$$\sum_{i \in \mathcal{N}_l^*} p_i \gamma_i \ln(\frac{\gamma_i}{\lambda_i} p_i) - B_l = 0 \quad (35)$$

Now we formulate the dual problem.
By substituting Eq.(33) to the Lagrangian Eq.(28), we have

the objective function of the dual problem as follows.

$$
\begin{aligned}
&D(\alpha,\beta) = L(\boldsymbol{p}^*,\alpha,\beta)\\
=&-f(\boldsymbol{p}^*) + \alpha\Big[\sum_{i\in\mathcal{N}_l^*} p_i^*\gamma_i \ln(\frac{\gamma_i}{\lambda_i}p_i^*) - B_l\Big] - \sum_{i\in\mathcal{N}_l^*}\beta_i p_i^*\\
=&-\sum_{i\in\mathcal{N}_l^*}\gamma_i \ln(\frac{\gamma_i}{\lambda_i}h_i'(\alpha))\\
&+\alpha\Big[\sum_{i\in\mathcal{N}_l^*} h_i'(\alpha)\gamma_i \ln(\frac{\gamma_i}{\lambda_i}h_i'(\alpha)) - B_l\Big]
\end{aligned} \tag{36}
$$

The formal dual problem is then defined as follows.

$$
\begin{aligned}
&\max D(\alpha,\beta)\\
&\text{s.t.}\quad \alpha \geq 0\\
&\qquad \beta_i \geq 0, \forall i \in \mathcal{N}_l^*
\end{aligned} \tag{37}
$$

Since we have confirmed that $\beta_i = 0$, then the dual problem is transformed to the following one.

$$
\begin{aligned}
&\max D(\alpha,0)\\
&\text{s.t.}\quad \alpha \geq 0
\end{aligned} \tag{38}
$$

The first order differentiation with respect to α of $D(\alpha,0)$ is as follows.

$$
\begin{aligned}
\frac{\partial D(\alpha,0)}{\partial\alpha} =& \sum_{i\in\mathcal{N}_l^*}\gamma_i h_i'(\alpha)\Big[\alpha - \frac{1}{h_i(\alpha)}\Big]\\
&+\sum_{i\in\mathcal{N}_l^*} h_i(\alpha)\gamma_i \ln(\frac{\gamma_i}{\lambda_i}h_i(\alpha)) - B_l\\
&+\alpha\Big[\sum_{i\in\mathcal{N}_l^*}\Big[h_i'(\alpha)\gamma_i \ln(\frac{\gamma_i}{\lambda_i}h_i(\alpha))\Big]\Big]
\end{aligned} \tag{39}
$$

By substituting Eq.(33) and Eq.(34), we have

$$
\begin{aligned}
\frac{\partial D(\alpha,0)}{\partial\alpha} =& -B_l + \sum_{i\in\mathcal{N}_l^*}\frac{\gamma_i[W(\frac{\gamma_i e}{\lambda_i\alpha})-1]}{\alpha[1+W(\frac{\gamma_i e}{\lambda_i\alpha})]}\\
&+\sum_{i\in\mathcal{N}_l^*}\Big[\frac{\gamma_i}{\alpha W(\frac{\gamma_i e}{\lambda_i\alpha})[1+W(\frac{\gamma_i e}{\lambda_i\alpha})]}\ln(\frac{\gamma_i}{\lambda_i}\cdot\frac{1}{\alpha W(\frac{\gamma_i e}{\lambda_i\alpha})})\Big]
\end{aligned} \tag{40}
$$

We define a function $G(\alpha)$ as follows.

$$
\begin{aligned}
G(\alpha) =& \sum_{i\in\mathcal{N}_l^*}\frac{\gamma_i[W(\frac{\gamma_i e}{\lambda_i\alpha})-1]}{\alpha[1+W(\frac{\gamma_i e}{\lambda_i\alpha})]}\\
&+\sum_{i\in\mathcal{N}_l^*}\Big[\frac{\gamma_i}{\alpha W(\frac{\gamma_i e}{\lambda_i\alpha})[1+W(\frac{\gamma_i e}{\lambda_i\alpha})]}\ln(\frac{\gamma_i}{\lambda_i}\cdot\frac{1}{\alpha W(\frac{\gamma_i e}{\lambda_i\alpha})})\Big]
\end{aligned} \tag{41}
$$

The point α^* satisfy the following Eq.(42) is the optimal solution for the dual problem.

$$
G(\alpha) = B_l \tag{42}
$$

It is difficult to get the explicit solution for α^*, we will get the optimal solution α^* as well as p_i^* by numeric analysis.